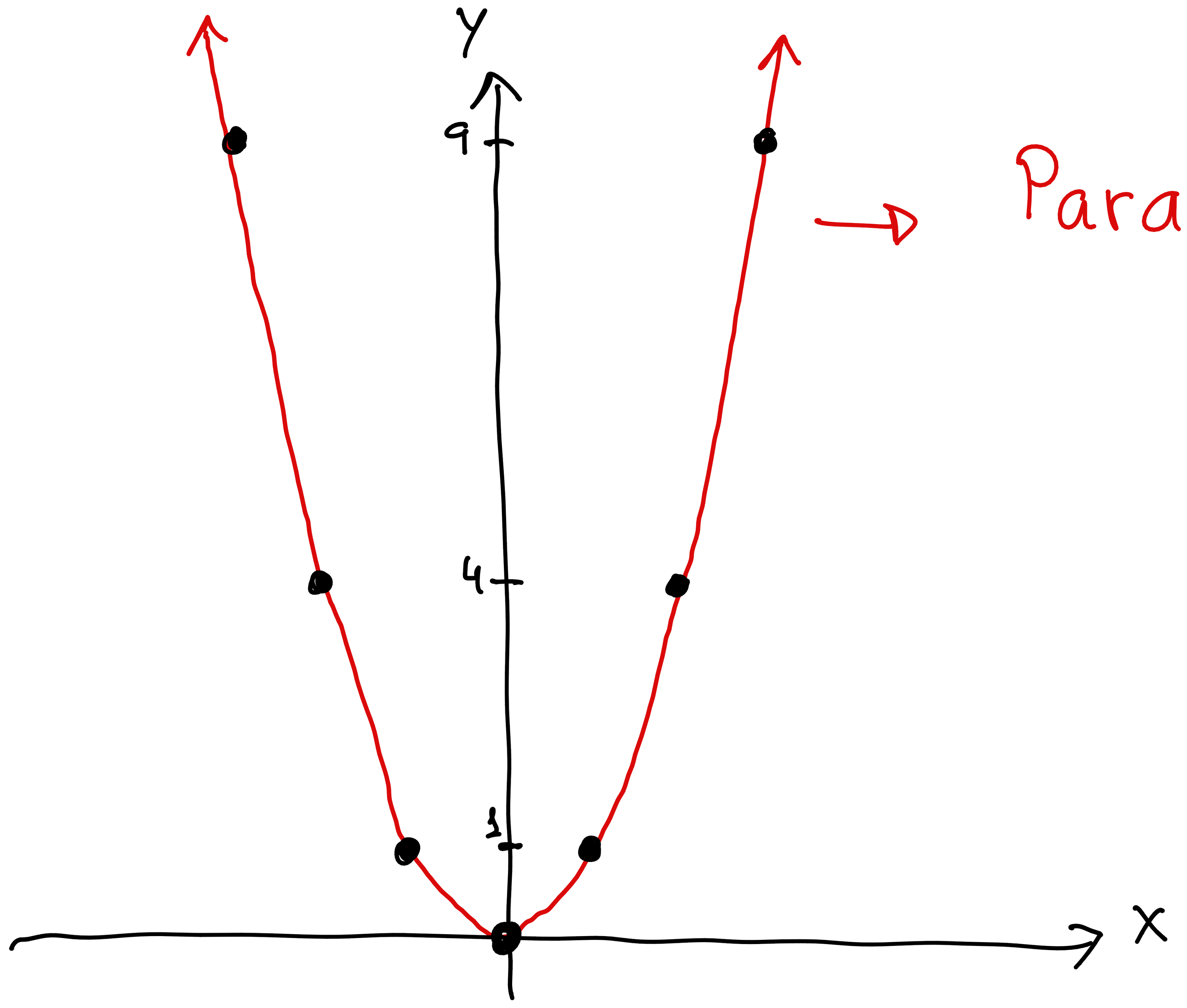
Class 14 - §4.1 & §4.2 Quadratic Functions

Review

$$\boxed{f(x) = ax^2+bx+c}$$

→ Basic Function $f(x) = x^2$, $y = f(x)$

x	y
0	0
−1	1
+1	1
−2	4
+2	4
+3	9
−3	9



→ Parabola

→ Evaluate Functions:

Evaluate $f(x) = (x-3)^2 - 5$ at $x = \frac{1}{2}$: $\left(\frac{1}{2}-3\right)^2 - 5 = \left(-\frac{5}{2}\right)^2 - 5 = \frac{5}{4}$

$8x^2 + 11$ at $x = \frac{1}{4}$: $8\left(\frac{1}{4}\right)^2 + 11 = \frac{1}{2} + 11 = \frac{23}{2}$.

→ Solve quadratic equation $ax^2+bx+c=0$

1) $x^2 - 4x - 5 = 0 \rightarrow (x+1)(x-5) = 0 \rightarrow x = -1,\ x = 5$

2) $\overset{a}{1}x^2 \overset{b}{-2}x \overset{c}{-5} = 0 \rightarrow$ quad. formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

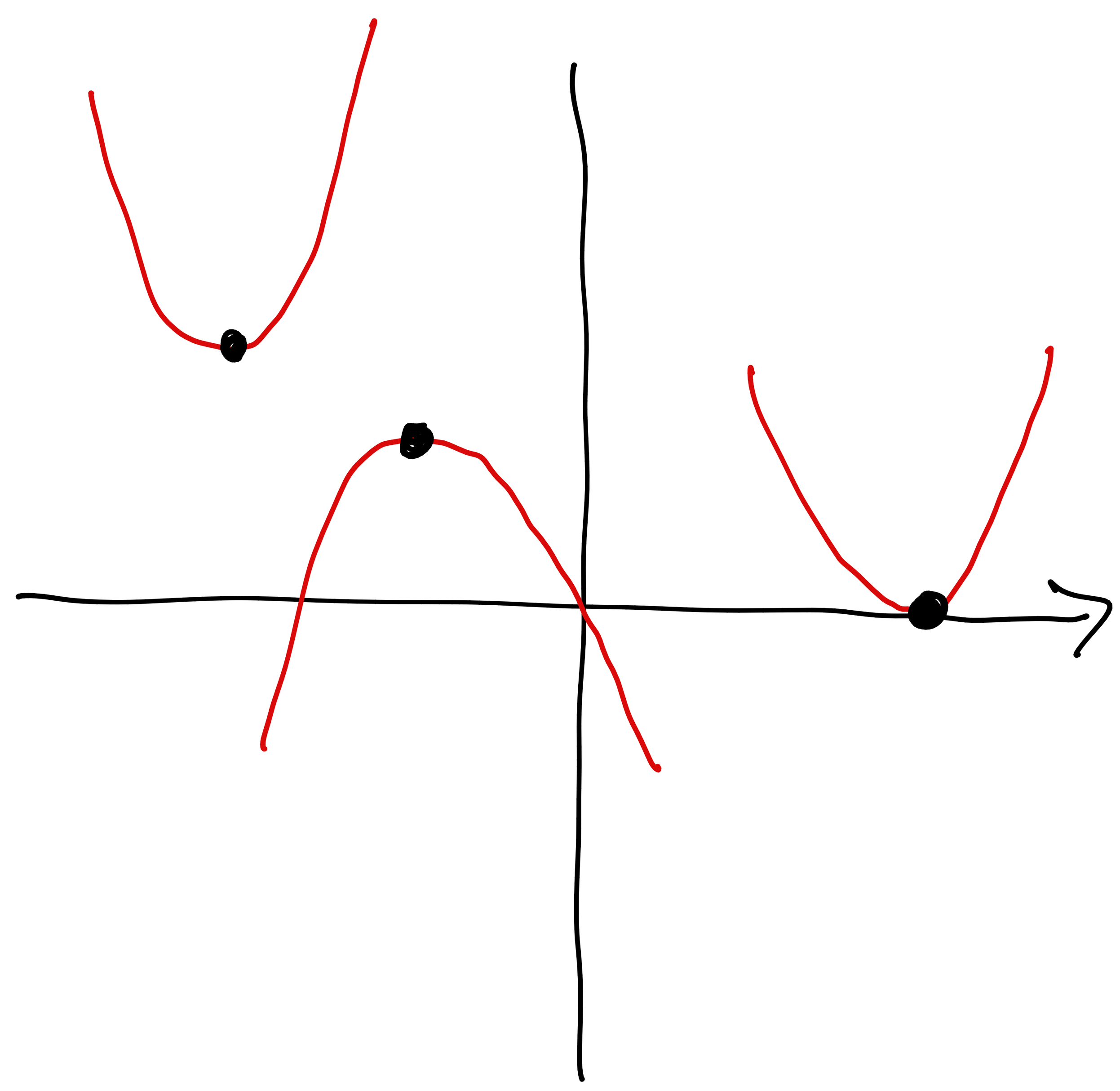
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot1\cdot(-5)}}{2\cdot1} = \frac{2 \pm \sqrt{4+20}}{2} = \frac{2 \pm \sqrt{24}}{2}$$

$\sqrt{24} = \sqrt{4\cdot6} = \sqrt{4}\cdot\sqrt{6} = 2\sqrt{6}$

$$= \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}.$$

$f(x) = ax^2 + bx + c \longrightarrow a(x-h)^2 + k$

Standard form → Vertex form

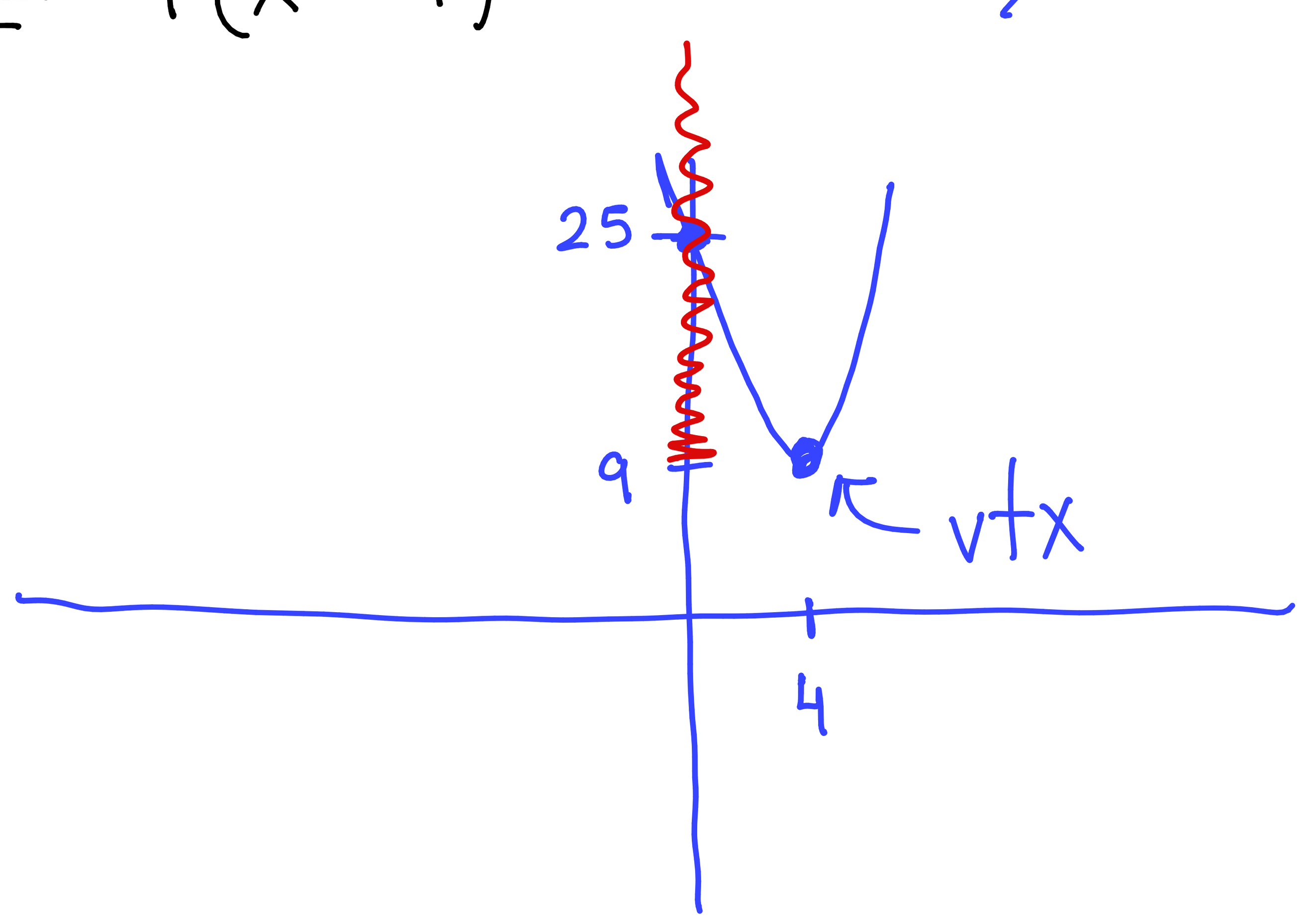


- vertex $= (h, k)$

$a > 0 \rightarrow$ "opens up"

$a < 0 \rightarrow$ "opens down"

Ex1: $1 \cdot (x-4)^2 + 9 \rightarrow$ Graph: and vertex is $(4, 9)$



rel. min. at $x = 4$ that has value $y = 9$

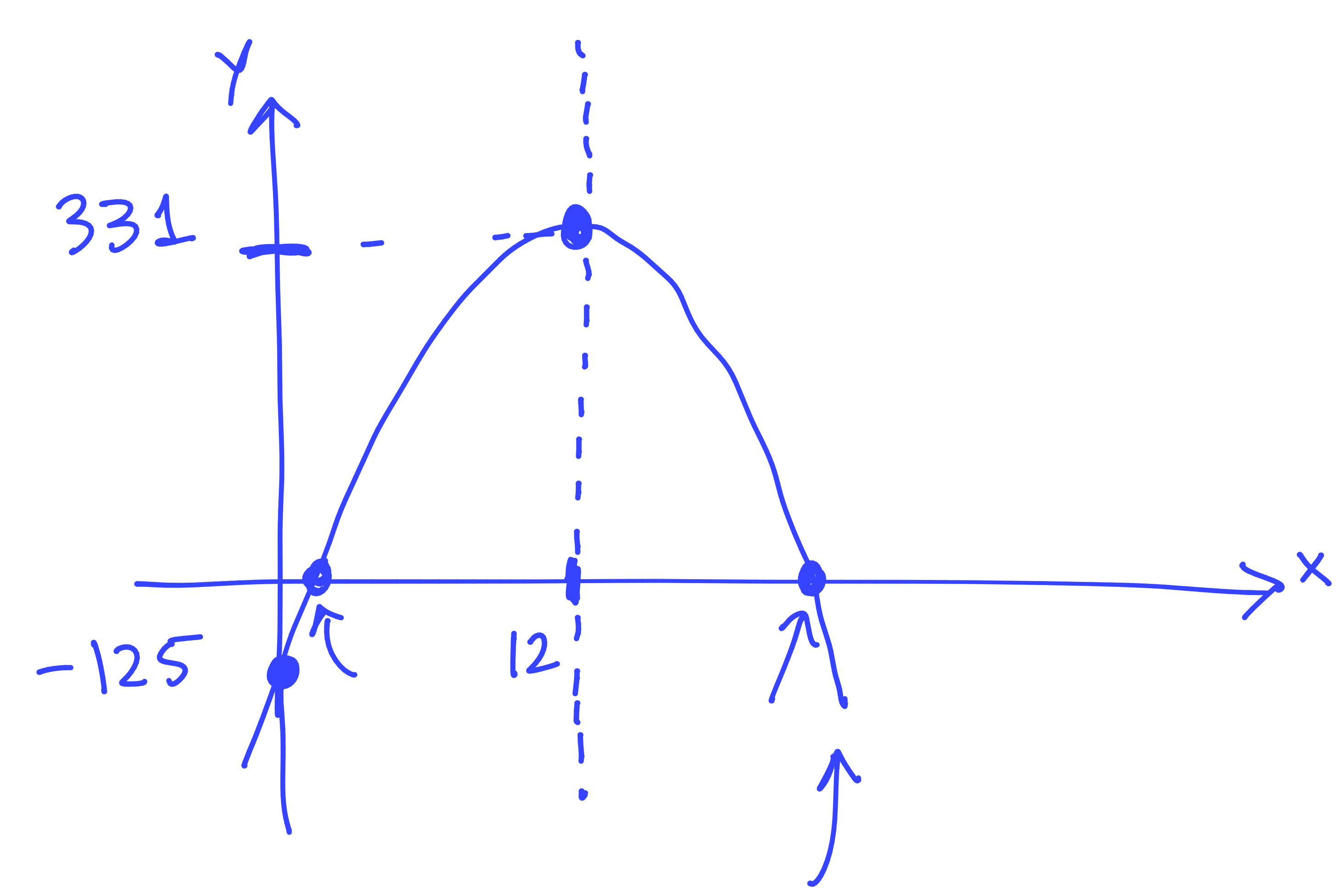
Domain: $(-\infty, \infty)$

Range: $a > 0$: $[9, \infty)$

→ Simplify some expressions:

Ex1: $(6y + 7) - (-y^2 + 6y + 7)$

$\cancel{6y} + \cancel{7} + y^2 - \cancel{6y} - \cancel{7} = y^2$



Ex2: Revenue $R(x) = 80x - 3x^2$

Cost $C(x) = 125 + 6x$

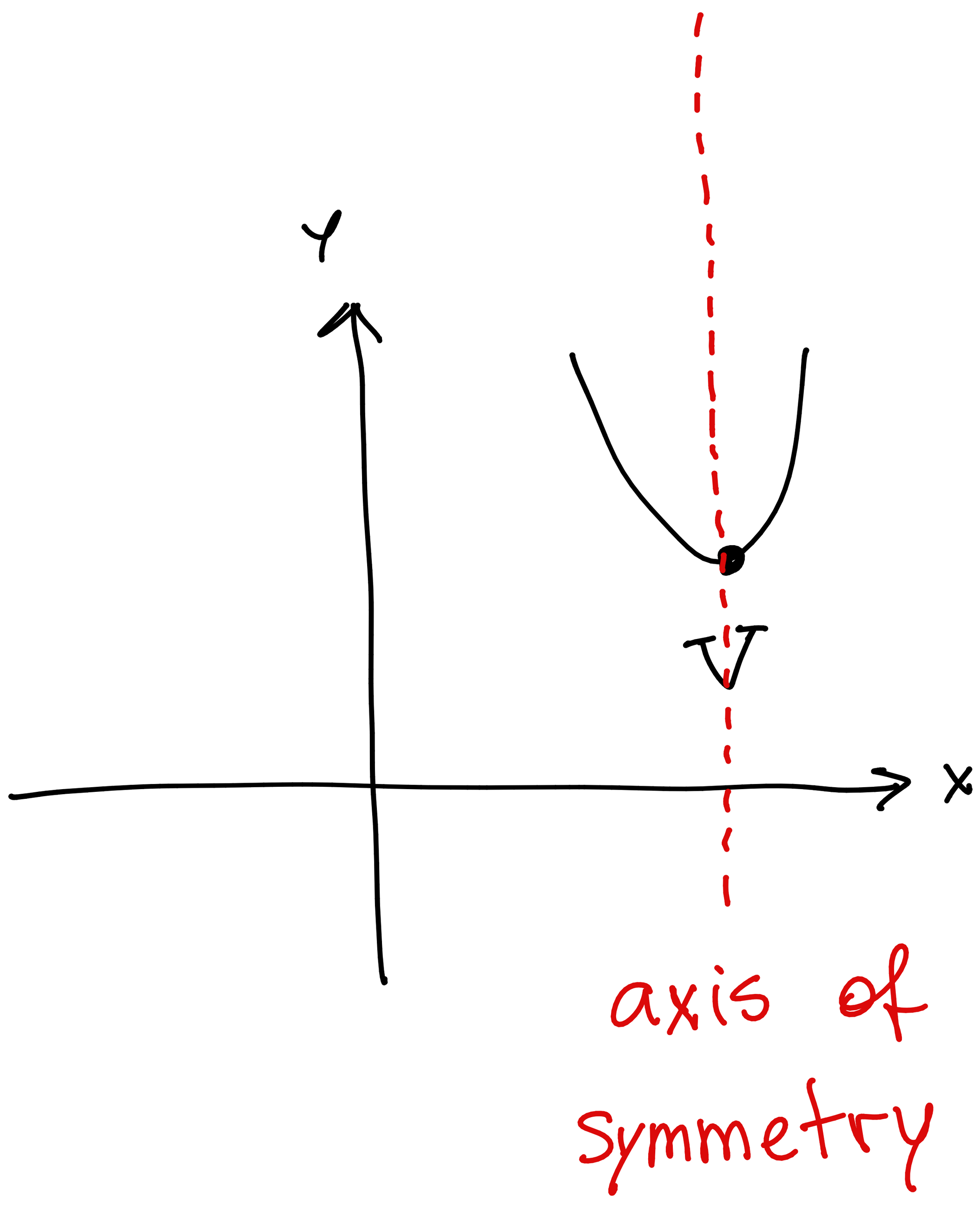
$P(x) = R(x) - C(x)$

$= (80x - 3x^2) - (125 + 6x)$

$= -3x^2 + 74x - 125.$

Quadratic Functions

$f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$



$V = (h, k)$ or $V = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$x = h$ or $x = -\frac{b}{2a}$

x & y-intercepts:

↳ $f(x) = 0$ & solve

$x = 0$ & solve

Domain & Range

Domain: $(-\infty, \infty)$

Range:

↳ $a > 0$: $[k, \infty)$ or $\left[f\left(-\frac{b}{2a}\right), \infty\right)$

$a < 0$: $(-\infty, k]$ or $\left(-\infty, f\left(-\frac{b}{2a}\right)\right)$

Ex 1: Consider the function $f(x) = \underbrace{\boxed{2}}_{a}(x-3)^2 - 10$.

Find:

Does the parabola open up or down? $a = 2 > 0$. UP

	Method	Answer
a) Vertex →	Look at (h, k)	$(3, -10)$
b) Axis of Symmetry →	$x = h$	$x = 3$
c) Intercepts →	$x = 0$ & $y = 0$	$y = 8$, $x = 3 \pm \sqrt{10}$
d) Domain & Range →	$(-\infty, \infty)$ & $(-\infty, k]$ or $[k, \infty)$	$[-10, \infty)$

c) Intercepts: $x=0$: $y=2(0-3)^2-10 = 2\cdot(-3)^2-10 = 2\cdot 9 - 10 = 8$.

$y=0$: $\quad 0=(x-3)^2-10 \rightarrow \sqrt{10}=\sqrt{(x-3)^2} \rightarrow \pm\sqrt{10}=x-3 \rightarrow 3\pm\sqrt{10}=x$

$\quad +10 \qquad\qquad +10 \qquad\qquad\qquad +3 \qquad +3$

Ex2: Consider the function $g(x)=x^2-7x+12$. $\leftrightarrow ax^2+bx+c$

a) The parabola opens up or down? $a>0$ or $a<0 \Rightarrow$ UP

b) Symmetry axis? $x=-\frac{b}{2a} \Rightarrow x=-\frac{(-7)}{2\cdot 1}=\frac{7}{2}$.

c) Vertex? $\left(-\frac{b}{2a}, g\left(-\frac{b}{2a}\right)\right) \Rightarrow \left(\frac{7}{2}, g\left(\frac{7}{2}\right)\right)=\left(\frac{7}{2},-\frac{1}{4}\right)$

d) Intercepts? $x=0$ & $g(x)=0 \Rightarrow y=12$,

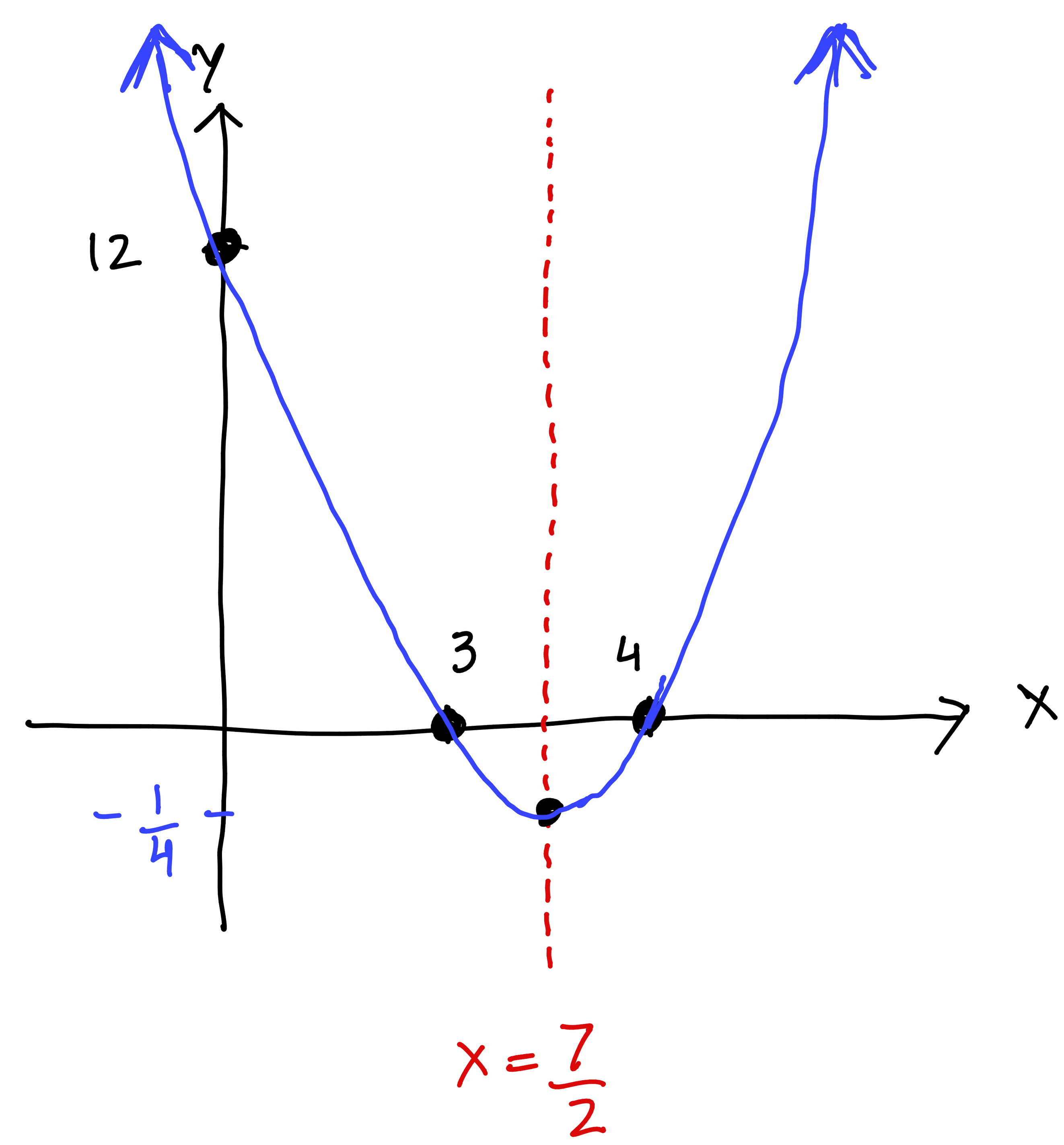
e) Graph of $g(x)$? Use the vertex & intercepts to sketch

$x^2-7x+12=0$

sum $\quad$ prod

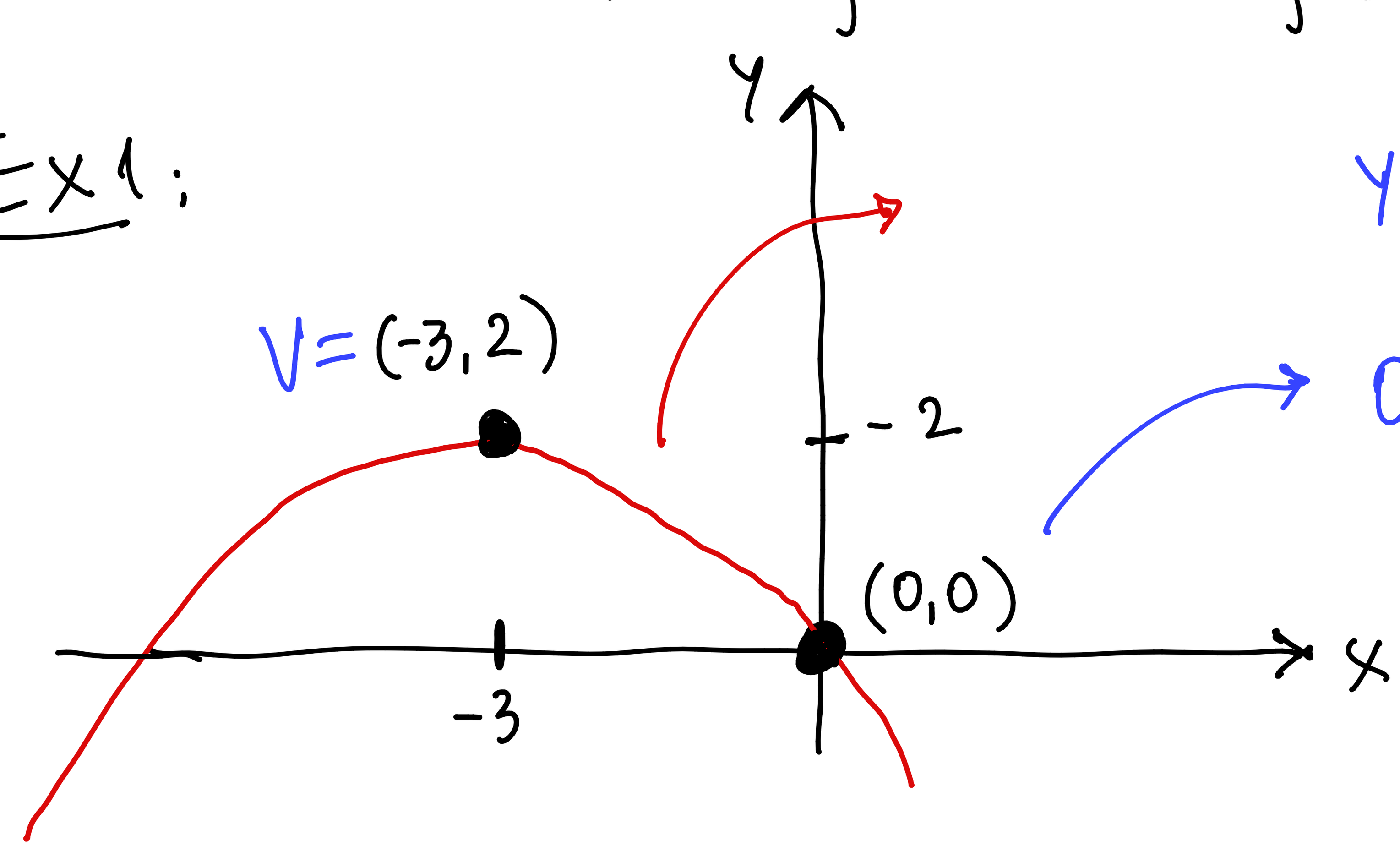
$(x-3)(x-4)=0$

$x=3$ or $x=4$



Determine the function from the graph: (ax^2+bx+c)

Ex1:

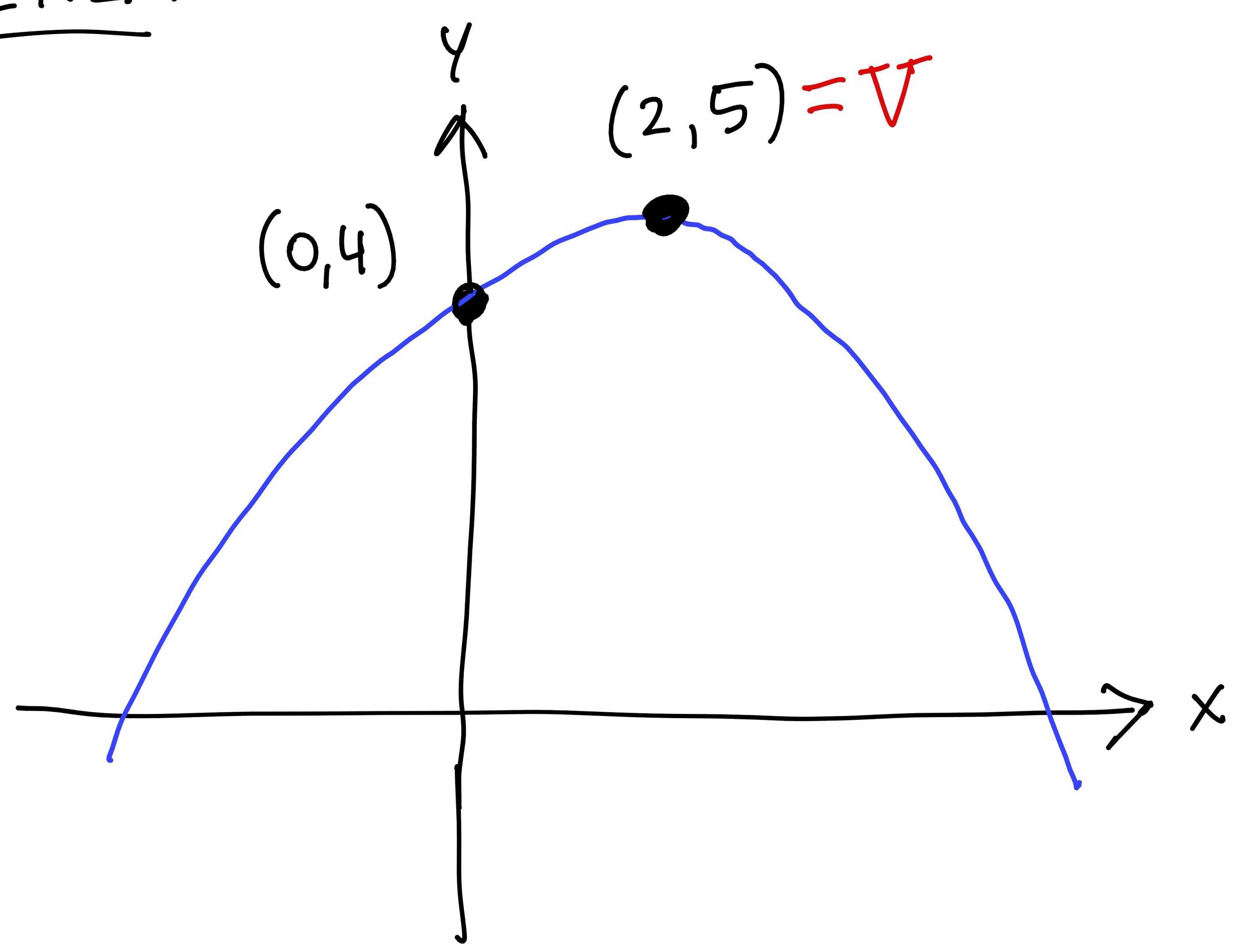


$y=a(x-h)^2+k \rightarrow y=a(x+3)^2+2$

$0=a(0+3)^2+2 \Rightarrow 0=9a+2 \Rightarrow a=-\frac{2}{9}$

$y=-\frac{2}{9}\underbrace{(x+3)^2}_{x^2+6x+9}+2 \Rightarrow \boxed{y=-\frac{2}{9}x^2-\frac{4}{3}x}$.

Ex2 :



1^{st} : write the vertex form

$y = a(x-h)^2 + k \longrightarrow y = a(x-2)^2 + 5$

2^{nd} : find "a"

$a = -\frac{1}{4} \longrightarrow y = -\frac{1}{4}(x-2)^2 + 5$

3^{rd} : get std form equation

$y = -\frac{1}{4}x^2 + \frac{1}{2} + 4$

2^{nd} : Plugin the second point (not the vertex)

$$y = a(x-2)^2 + 5 \xrightarrow[(0,4)]{} 4 = a(0-2)^2 + 5 \longrightarrow \underset{-5}{4} = 4a \underset{-5}{+5} \longrightarrow \boxed{a = -\frac{1}{4}}$$

3^{rd} : Vertex form → Standard form (expand the square term)

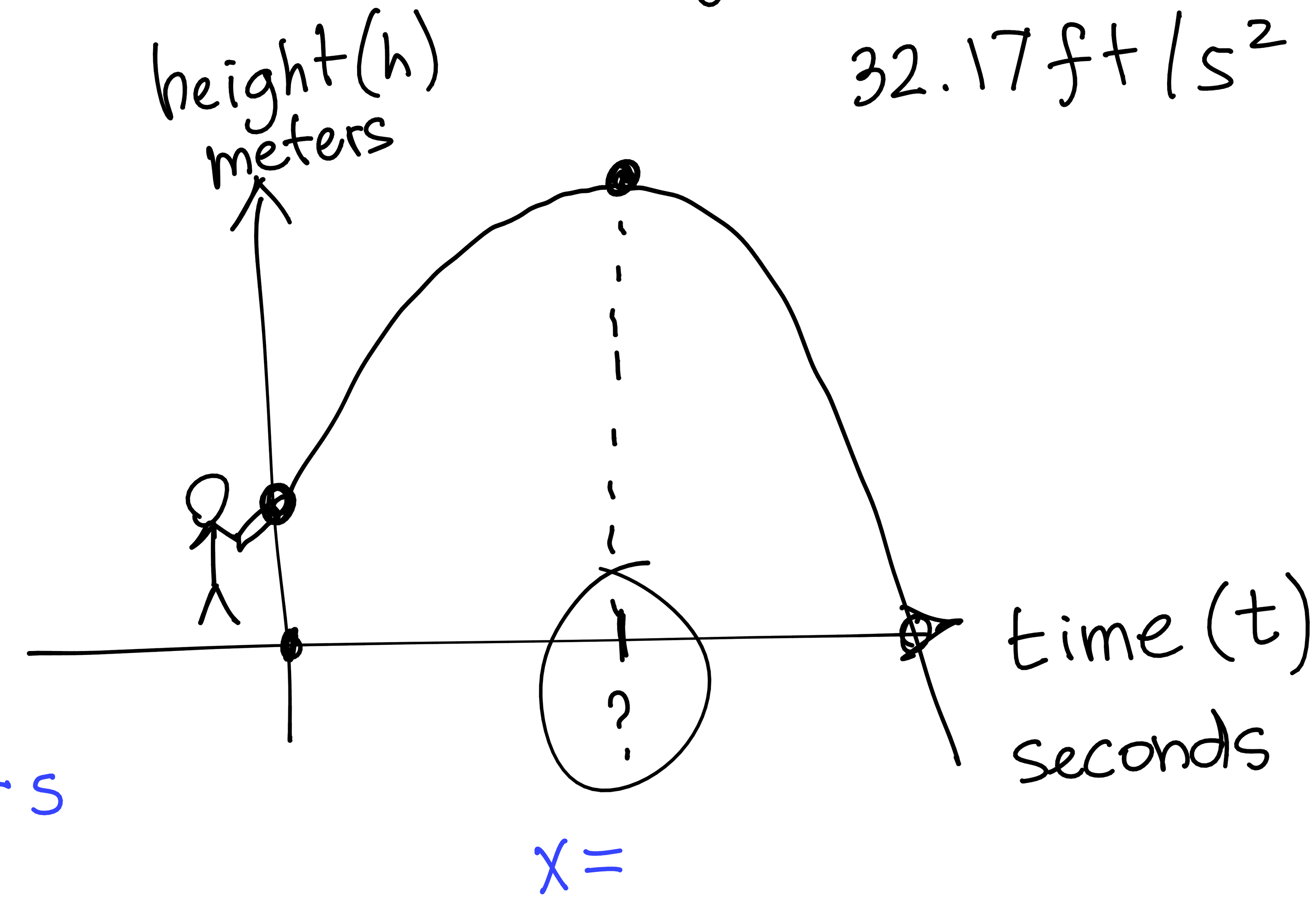
$$y = -\frac{1}{4}(x-2)^2 + 5 \longrightarrow y = -\frac{1}{4}(x^2 - 2x + 4) + 5 \longrightarrow y = -\frac{1}{4}x^2 + \frac{1}{2}x - 1 + 5$$

$$\boxed{y = -\frac{1}{4}x^2 + \frac{1}{2}x + 4}$$

Applications

Ex 1: $h(t) = -4.9t^2 + 39.2t + 1$

$g \approx 9.8\ m/s^2$ or $32.17\ ft/s^2$



a) How long does it take for the ball to reach the max. height?

$t = ?$ (symm. axis)

$t = -\frac{b}{2a} \Rightarrow t = -\frac{39.2}{2\cdot(-4.9)} = 4s$

b) What is the maximum height?

$h(4) = -4.9(4)^2 + 39.2(4) + 1 = 79.4\,m$

Ex 2: x is the # of barrels of oil on hand (in millions)

Demand equation: $p = -\frac{1}{4}x + 170.$

a) What should be charged for a barrel if there are 2 million on hand?

$x = 2$, what is $p = ?$ $\quad p = -\frac{1}{4}\cdot 2 + 170 = 169.50$

b) What quantity maximizes the revenue? What is max. revenue?

$R(x) = xp \Rightarrow R(x) = x\left(-\frac{1}{4}x + 170\right) \Rightarrow R(x) = -\frac{1}{4}x^2 + 170x.$

$V = ?$ $\quad x = -\frac{b}{2a} \Rightarrow x = -\frac{170}{2\cdot(-\frac{1}{4})} \Rightarrow x = 340 \Rightarrow R(340) = -28{,}900 + 57{,}800$

$= 28{,}900$

c) What price should be charged in order to maximize the revenue?

max revenue at $x = 340 \Rightarrow p(340) = -\frac{1}{4}\cdot 340 + 170 \Rightarrow p = 85.$

Ex3: "x" is the amount of vans to sell.

Demand equation: $p = -\frac{1}{40} \cdot x + 10\,000$.

Cost equation: $C(x) = 6250x + 20000$.

a) $R(x) = ?$ $R(x) = xp \Rightarrow R(x) = -\frac{1}{40}x^2 + 10000x$

b) $P(x) = ?$ $P = R - C \Rightarrow P(x) = \left(-\frac{1}{40}x^2 + 10000x\right) - (6250x + 20000)$

$\Rightarrow P(x) = -\frac{1}{40}x^2 + 3750x - 20\,000$.

c) Find the amount of vans "x" that maximizes the profit. What is the profit? (Vertex of the profit function)

$x = -\frac{b}{2a} \Rightarrow x = -\frac{3750}{2 \cdot \left(-\frac{1}{40}\right)} = -\frac{3750}{-\frac{\cancel{2}}{\cancel{40}}\,\frac{1}{20}} = 3750 \cdot \frac{20}{1} = 75000$.

$y = P(75000) = -\frac{1}{40}(75000)^2 + 3750(75000) - 20000$

$= -140\,625\,000 + 281\,250\,000 - 20000$

$= 140\,605\,000$.

d) What price should be charged for each van?

$p = -\frac{1}{40}x + 10000 \longrightarrow x = 75000$

$p = -\frac{1}{40}(75000) + 10000 \Rightarrow p = -1875 + 10000 \Rightarrow p = 8125$.